Solutions - Homework 4

(Due date: June 16th)

Presentation and clarity are very important! Show your procedure!

PROBLEM 1 (20 PTS)

- Calculate the result (provide the 32-bit result) of the following operations with 32-bit floating point numbers. Truncate the results when required. When doing fixed-point division, use x = 4 fractional bits. Show your procedure.

✓ C3FA8000 - C1E00000	✓ D0D80000 + D0FA0000	✓ 80C00000×FAD00000	✓ 7B380000 ÷ C8A00000

✓ X = C3FA8000 - C1E00000:

C3FA8000: 1100 0011 1111 1010 1000 0000 0000 0000

$e + bias = 10000111 = 135 \rightarrow e = 135 - 127 = 8$ $\quad Significand = 1.11110101$

C3FA8000 = -1.11110101×2^8

C1E00000: 1100 0001 1110 0000 0000 0000 0000 0000

$e + bias = 10000011 = 131 \rightarrow e = 131 - 127 = 4$ $\quad Significand = 1.11$

C1E00000 = -1.11×2^4

$X = -1.11110101 \times 2^8 + 1.11 \times 2^4 = -1.11110101 \times 2^8 + \frac{1.11}{2^4} \times 2^8$

$X = -(1.11110101 - 0.000111) \times 2^8$

We perform unsigned subtraction: $X = -1.11011001 \times 2^8$

b_9=0	b_8=0	b_7=0	b_6=0	b_5=1	b_4=1	b_3=0	b_2=0	b_1=0	b_0=0	
	1.1	1	1	1	0	1	0	1		−
	0.0	0	0	1	1	1	0	0		
	1.1	1	0	1	1	0	0	1		

$X = -1.11011001 \times 2^8$, $e + bias = 8 + 127 = 135 = 10000111$

X = 1100 0011 1110 1100 1000 0000 0000 0000 = C3EC8000

✓ X = D0D80000 + D0FA0000:

D0D80000: 1101 0000 1101 1000 0000 0000 0000 0000

$e + bias = 10100001 = 161 \rightarrow e = 161 - 127 = 34$ $\quad Significand = 1.1011$

D0D80000 = -1.1011×2^{34}

D0FA0000: 1101 0000 1111 1010 0000 0000 0000 0000

$e + bias = 10100001 = 161 \rightarrow e = 161 - 127 = 34$ $\quad Significand = 1.1111010$

D0FA0000 = -1.1111010×2^{34}

$X = -1.1011 \times 2^{34} - 1.1111010 \times 2^{34}$ (unsigned addition)

c_7=1	c_6=1	c_5=1	c_4=1	c_3=1	c_2=0	c_1=0	c_0=0	
	1.1	0	1	1	0	0		+
	1.1	1	1	1	0	1		
1	1.1	0	1	0	0	1		

$X = -11.101001 \times 2^{34} = -1.1101001 \times 2^{35}$

$e + bias = 35 + 127 = 162 = 10100010$

X = 1101 0001 0110 1001 0000 0000 0000 0000 = D1690000

✓ X = 80C00000 × FAD00000:

80C00000: 1000 0000 1100 0000 0000 0000 0000 0000

$e + bias = 00000001 = 1 \rightarrow e = 1 - 127 = -126$ $\quad Significand = 1.1$

80C00000 = $= -1.1 \times 2^{-126}$

FAD00000: 1111 1010 1101 0000 0000 0000 0000 0000

$e + bias = 11110101 = 245 \rightarrow e = 245 - 127 = 118$ $\quad Significand = 1.101$

FAD00000 = -1.101×2^{118}

		1.1	0	1 x
			1.1	
		1	1	0 1
	1	1	0	1
1	0	0.0	1	1 1

$X = (-1.1 \times 2^{-126}) \times (-1.101 \times 2^{118}) = 10.0111 \times 2^{-8} = 1.00111 \times 2^{-7}$

$e + bias = -7 + 127 = 120 = 01111000$

X = 0011 1100 0001 1100 0000 0000 0000 0000 = 3C1C0000

✓ X = 7B380000 ÷ C8A00000:

7B380000: 0111 1011 0011 1000 0000 0000 0000 0000

$e + bias = 11110110 = 246 \rightarrow e = 246 - 127 = 119$ $\quad Significand = 1.0111$

7B380000 = 1.0111×2^{119}

C8A00000: 1100 1000 1010 0000 0000 0000 0000 0000

$e + bias = 10010001 = 145 \rightarrow e = 145 - 127 = 18$ $\quad Significand = 1.01$

C8A00000 = -1.01×2^{18}

$$X = -\frac{1.0111 \times 2^{119}}{1.01 \times 2^{18}} = -\frac{1.0111}{1.01} \times 2^{101}$$

```
          000010010
10100 ) 101110000
        10100
        -----
          11000
          10100
          -----
           1000
```

Alignment:
$\frac{1.0111}{1.01} = \frac{1.0111}{1.0100} = \frac{10111}{10100}$

Append $x = 4$ zeros: $\frac{\mathbf{10111}\mathbf{0000}}{10100}$

Integer division:
$Q = 10010 \rightarrow Qf = 1.001$

Thus: $X = -1.001 \times 2^{101}$, $e + bias = 101 + 127 = 228 = 11100100$
X = 1111 0010 0001 0000 0000 0000 0000 0000 = F2100000

PROBLEM 2 (20 PTS)

- Complete the table for the following DFX formats (10 pts)

$B = 2^{n-p_0-2}$
$num0 \in [-2^{n-p_0-2}, 2^{n-p_0-2} - 2^{-p_0}]$
$num1 \in [-2^{n-p_1-2}, -2^{n-p_0-2} - 2^{-p_1}] \cup [2^{n-p_0-2}, 2^{n-p_1-2} - 2^{-p_1}]$
$Dynamic\ Range = 20\log_{10}(2^{n-2-p_1+p_0})$

DFX format	p_0	p_1	Number of bits of significand	Boundary value	$num0$ range	$num1$ range	Dynamic Range (dB)
8_4_2	4	2	7	4	[-4,3.9375]	[-16,-4.25] U [4,15.75]	48.1648
16_8_4	8	4	15	64	[-64, 63.99609375]	[-1024, -64.0625] U [64, 1023.9375]	108.3708

- Convert the following signed fixed-point numbers in format [16 8] to the dual fixed-point format 16_8_4. (10 pts)

FX	3A.CD	9B.E6	7A.CE	CA.FE
DFX				

✓ 3A.CD:
0011 1010.1100 1101 ⇒ To DFX 16_8_4 ($num0$): 0011101011001101 = 3ACD

✓ 9B.E6:
1001 1011.1110 0110 ⇒ To DFX 16_8_4 ($num0$): 0001101111100110 = not a $num0$!
⇒ To DFX 16_8_4 ($num1$): 1111100110111110 = F9BE

✓ 7A.CE:
0111 1010.1100 1110 ⇒ To DFX 16_8_4 ($num0$): 0111101011001110 = not a $num0$!
⇒ To DFX 16_8_4 ($num1$): 1000011110101100 = 87AC

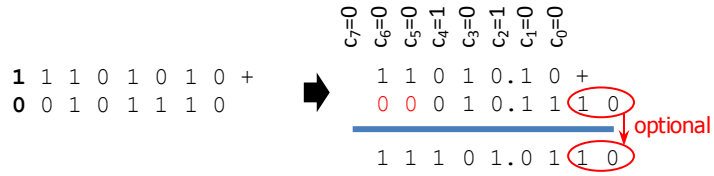
✓ CA.FE:
1100 1010.1111 1110 ⇒ To DFX 16_8_4 ($num0$): 0100101011111110 = 4AFE

PROBLEM 3 (20 PTS)

- Calculate the result of the following operations where the numbers are represented in dual fixed-point arithmetic. Note that the results must be in the same format. Include an overflow bit when necessary.

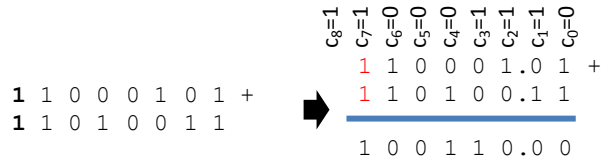
DFX Format: 8_4_2	Result	overflow		Result	overflow
EA+2E			EB-99		
D3+C5			65+FD		

✓ EA+2E:



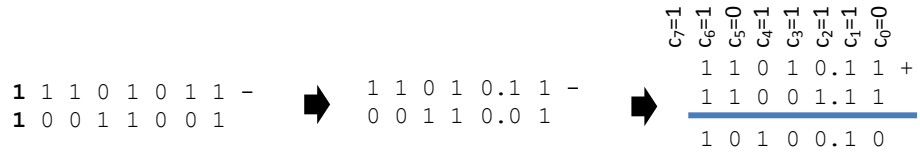
11101.0110 ⇒ To DFX 8_4_2 (*num*0): 0101.0110 = 56 Overflow = 0

✓ D3 + C5:



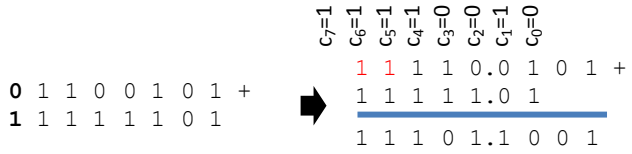
100110.01 ⇒ To DFX 8_4_2 (*num*0): 01100000 = not a *num*0!
⇒ To DFX 8_4_2 (*num*1): 10011000 = not a *num*1! Overflow = 1

✓ EB-99:



10100.10 ⇒ To DFX 8_4_2 (*num*0): 01001000 = not a *num*0!
⇒ To DFX 8_4_2 (*num*1): 11010010 = D2 Overflow = 0

✓ 65+FD:



11101.1001 ⇒ To DFX 8_4_2 (*num*0): 01011001 = 59 Overflow = 0

PROBLEM 4 (40 PTS)

- Attach your Project Status Report (no more than 3 pages, single-spaced, 2 columns, only one submission per group). This report should contain the current status of your project. For formatting, use the provided template (`Final Project - Report Template.docx`). More details need to be provided:
  - ✓ Details, i.e., architecture of the AXI Interface
  - ✓ Allocation of tasks: i) software routine, and ii) reconfigurable hardware.
    - Software routine: provide top-level pseudo-code of your software application
    - If you plan to use run-time alterable hardware, indicate what tasks it will be doing.
  - ✓ Hardware Architecture: Include a Block Diagram with a complete I/O description (how many signals, how many bits per signal) and I/O mechanism.